ANALYSIS III FINAL EXAMINATION

Total marks: 60
Date: 13th November, 2017
Time: 2-5 pm

(1) Interpret the iterated integral $\int_0^1 \int_{x^2}^x (x^2 + y^2)\, dydx$ as an integral over a certain Jordan region in $\mathbb{R}^2$ and compute this integral. This integral is equal to a certain iterated integral, first with respect to x, and then with respect to y. Describe and evaluate that integral. State the theorem which implies the equality of these two integrals.(5+5+5=15 marks)

(2) Let $A = \{(x, y) \in \mathbb{R}^2 | x \geq 0, y \geq 0, x^2 + y^2 \leq 4, x^2 - y^2 \geq 1\}$. Compute $\int_A xy\, dV(x, y)$, by making a change of variables $u = x^2 + y^2, v = x^2 - y^2$ for $x \geq 0, y \geq 0$. State the theorem which is used. (8+7=15 marks)

(3) Let C be the curve obtained by intersecting (in $\mathbb{R}^3$) the plane $x = z$ with the cylinder $x^2 + y^2 = 1$, oriented anticlockwise when viewed from above (positive z-axis). Let S be the region inside this curve, oriented with the upward pointing normal. Let $F = (x, z, 2y)$ be the component vector field of a 1-form on $\mathbb{R}^3$. Compute both sides of Stokes's theorem in this situation, and verify they are equal. (5+5 = 10 marks)

(4) Let $B := \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 \leq 1\}$ be the solid ball of radius less than or equal to 1 in $\mathbb{R}^3$. Use Gauss's theorem to compute the integral of the 2-form $(x - cos(y))dy \wedge dz + (xe^z - y)dz \wedge dx + (z - sin(2xy))dx \wedge dy$ over the boundary of B (which is the sphere S^2 of radius 1, with the normal vector assumed to be pointing outward at each point). (10 marks)

(5) For $n \in \mathbb{N}$, define $f_n(x) = \frac{x}{1+nx^2}$, for any $x \in \mathbb{R}$. Does the sequence of functions $\{f_n\}$ converge uniformly on $\mathbb{R}$? (10 marks)